System with Parallel Loops," *Journal of Heat Transfer*, Vol. 103, No. 4, 1981, pp. 645–652.

[8]Sen, M., and Fernandez, J. L., "One-Dimensional Modeling of Multiple-Loop Thermosyphons, *International Journal of Heat and Mass Transfer*, Vol. 28, No. 9, 1985, pp. 1788–1790.

[9]Sen, M., Pruzan, D. A., and Torrance, K. E., "Analytical and Experimental Study of Steady-State Convection in a Double-Loop Thermosyphon," *International Journal of Heat and Mass Transfer*, Vol. 31, No. 4, 1988, pp. 709–722.

[10]Salazar, O., Sen, M., and Ramos, E., "Flow in Conjugate Natural Circulation Loops," *Journal of Thermophysics and Heat Transfer*, Vol. 2, No. 2, 1988, pp. 180–183.

[11]Greif, R., "Natural Circulation Loops," *Journal of Heat Transfer*, Vol. 110, No. 4(B), 1988, pp. 1243–1258.

[12]Yilmaz, S. B., "Horizontal Shellside Thermosiphon Reboilers," *Chemical Engineering Progress*, Vol. 83, No. 11, 1987, pp. 64–70.

Heat Transfer to a Power Law Non-Newtonian Falling Liquid Film

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Nomenclature

B = width of plate
Br = Brinkman number
g = gravitational acceleration
Gz = Graetz number
Gz^* = modified Graetz number
h = heat transfer coefficient
K_f = thermal conductivity
n = power law index
Nu = Nusselt number
Pe = Peclet number
T = temperature
u = velocity component in x-direction
x, y = parallel and normal coordinates
ξ, ψ = dimensionless axial coordinates
μ = viscosity coefficient for power law fluids
α = thermal diffusivity
η, ϕ = dimensionless normal coordinates
δ = film thickness
θ = dimensionless temperature
ρ = density

Subscripts
i = inlet conditions
w = surface conditions

Superscript
$\bar{}$ = average conditions

Introduction

THIN falling film with heat transfer is important in modern technology. Non-Newtonian fluid falling film shell and tube exchangers are used in the food and polymer processing applications.

Experimental and theoretical studies of the heat transfer under thermally fully developed conditions were reported by Chun and Seban.[1,2] The conjugate heat transfer in falling liquid films has been recently studied by Gorla et al.[3,4] These investigations were concerned with Newtonian fluids. Murthy and Sarma[5] analyzed heat transfer in the entrance region of a non-Newtonian power law model laminar falling film under constant surface temperature conditions by means of an approximate integral method. The heat transfer from an isothermal inclined plate to non-Newtonian fluid falling films was studied by Stuckheli and Widmer.[6]

In the present work, we have studied the heat transfer in the thermal entrance of a laminar Ostwald-de-Waele type power law model non-Newtonian falling liquid film. The velocity field will be taken as fully developed, whereas the thermal field will be assumed to be developing. The effect of heat generation due to viscous dissipation is considered in the formulation of the problem.

Analysis

Consideration will be given to a vertical plate placed in a parallel stream of a hydrodynamically fully developed non-Newtonian laminar falling liquid film. The liquid flow is characterized by the power law rheological model. The total shear stress distribution in the liquid film is given by

$$\tau = \mu\left(\frac{du}{dy}\right)^n = \rho(\delta - y)g \tag{1}$$

Using the boundary condition of no slip at the wall and zero interfacial shear at the gas-liquid interface, one may obtain an expression for the velocity distribution in the following form:

$$\frac{u(\eta)}{U_0} = 1 - (1 - \eta)^{(n+1)/n} \tag{2}$$

where

$$\eta = (y/\delta); \qquad U_0 = \left(\frac{n}{n+1}\right)\left(\frac{\rho g}{\mu}\right)^{1/n}\delta^{(n+1)/n}$$

The governing energy equation may be written as

$$u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{du}{dy}\right)^{n+1} \tag{3}$$

with boundary conditions given by

$$x = 0\text{: } T = T_i \text{ (inlet condition)}$$

$$y = 0\text{: } q_w = -K_f\left(\frac{\partial T}{\partial y}\right)$$

$$y = \delta\text{: } \frac{\partial T}{\partial y} = 0 \text{ (zero interfacial heat flux)} \tag{4}$$

Proceeding with the analysis, define

$$\xi = \frac{x}{L}, \qquad \eta = \frac{y}{\delta}, \qquad \theta = \frac{T - T_i}{\left(\dfrac{q_w\,\delta}{K_f}\right)}$$

$$Pe = \frac{\rho CQ}{BK_f}, \qquad Gz = \frac{\delta}{L}Pe, \qquad Gz^* = Gz\left(\frac{2n+1}{n+1}\right)$$

$$Br = \frac{\mu(Q/B)^{n+1}[(2n+1)/n]^{n+1}}{Q_w\,\delta^{2n+1}}$$

$$\phi = \eta[2GZ^*/9\xi]^{1/3}, \qquad \psi = (9\xi/2Gz^*)^{1/3} \tag{5}$$

Received Dec. 6, 1989; revision received June 21, 1990; accepted for publication July 2, 1990.

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Substituting Eq. (5) expressions into Eqs. (3) and (4), one may write

$$\frac{3}{2\psi^3}\left[1 - (1 - \phi\psi)^{(n+1)/n}\right]\left(\phi\frac{\partial\theta}{\partial\phi} + \psi\frac{\partial\theta}{\partial\psi}\right) = \frac{1}{\psi^2}\frac{\partial^2\theta}{\partial\phi^2} + Br(1 - \phi\psi)^{(n+1)/n} \quad (6)$$

with boundary conditions given by

$$\phi = 0\colon \theta' = -1, \qquad \phi \to \infty\colon \theta \to 0 \quad (7)$$

Solutions and Results

A series solution will be sought for $\theta\ (\phi, \psi)$ in the form

$$\theta = \theta_0(\phi) + \psi\theta_1(\phi) + \psi^2\,\theta_2(\phi) + \ldots \quad (8)$$

Substituting Eq. (8) into Eqs. (6) and (7) and equating coefficients of like powers of ψ, we obtain a set of ordinary differential equations. These details are omitted in the interest of conserving space.

The local Nusselt number may be written as

$$Nu_x^* = \frac{h \times \delta}{K_f L\xi} = \left[\frac{1}{\theta_0(0) + \psi\theta_1(0) + \psi^2\,\theta_2(0) + \ldots}\right] \quad (9)$$

Table 1 Values of $_j(0)$ vs n and Br (j − 0, 1, 2, 3, and 4)

n	Br	$\theta_0(0)$	$\theta_1(0)$	$\theta_2(0)$	$\theta_3(0)$	$\theta_4(0)$
	0	0.619173	0.153971	0.043064	−0.006050	0.004381
0.2	1	0.619173	0.153971	0.201434	−0.128597	0.040025
	5	0.619173	0.153971	0.834733	−1.667363	0.217408
	0	0.708723	0.121546	0.030369	−0.006407	0.000030
1/3	1	0.708723	0.121546	0.237733	−0.136644	0.032693
	5	0.708723	0.121546	1.067132	−0.708848	0.163347
	0	0.892973	0.064611	0.014573	−0.004441	0.001639
1	1	0.892973	0.064611	0.343174	−0.163704	0.018090
	5	0.892973	0.064611	2.657516	−1.836402	0.084072
	0	1.022196	0.028276	0.006348	−0.002235	0.001043
3	1	1.022196	0.028276	0.436157	−0.190824	0.008374
	5	1.022196	0.028276	2.155453	−1.963181	0.037819
	0	1.058722	0.018263	0.004083	−0.001520	0.000745
5	1	1.058722	0.018263	0.464946	−0.019825	0.005513
	5	1.058722	0.018263	2.308220	−1.005083	0.024706

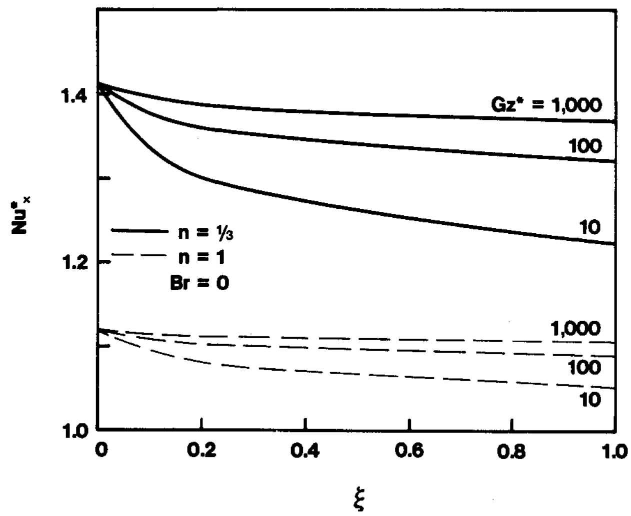

Fig. 1 Local Nusselt number vs the modified Graetz number (Br = 0).

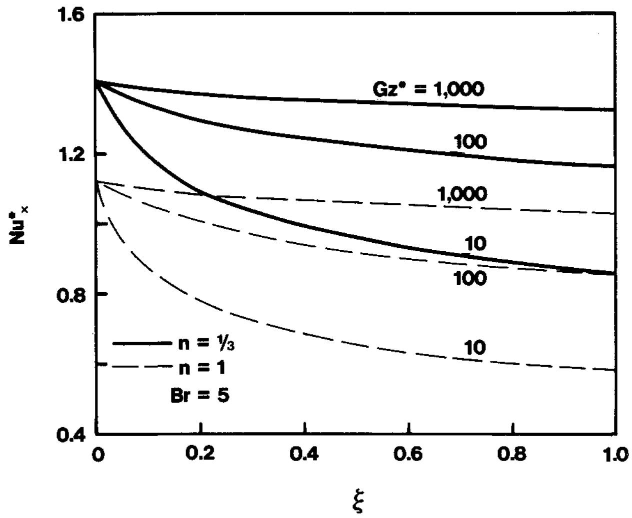

Fig. 2 Local Nusselt number vs the modified Graetz number (Br = 5).

In most of the practical applications, it is the surface characteristics, such as heat transfer rate, that are important. The values of $\theta_0(0)$, $\theta_1(0)$, $\theta_2(0)$, $\theta_3(0)$, and $\theta_4(0)$ proportional to the Nusselt number have been been tabulated in Table 1. With the information provided in Table 1, one may compute the local Nusselt number in a straightforward way from Eq. (9). We have displayed the variation of the local Nusselt number vs the modified Graetz number in Figs. 1 and 2. In these figures, the power law index n and the Brinkman number Br have been treated as prescribable parameters. The results indicate that the effect of viscous dissipation is to increase heat transfer rate. For a given value of the Graetz number, the heat transfer rate is higher in the case of pseudoplastic fluids ($n < 1$) than Newtonian ($n = 1$) fluids. For a given fluid, in general, the average heat transfer rate increases with the modified Graetz number.

Concluding Remarks

In this paper, we have studied the heat transfer in the thermal entrance region of an Ostwald-de-Waele type non-Newtonian laminar falling film. Numerical solutions were obtained for the temperature distribution within the film, as well as for the heat transfer to the film. The results indicate that the average Nusselt number increases with the modified Graetz number. For a given value of the modified Graetz number, the average heat transfer rate was found to be higher in the case of pseudoplastic fluids than dilatant fluids. The effect of viscous dissipation is to augment the heat transfer rate.

References

[1]Chun, K. R., and P. A. Seban, "Heat Transfer to Evaporating Liquid Films," *Journal of Heat Transfer*, Transactions of American Society of Mechanical Engineers, Vol. 93, No. 2, 1971, pp. 391–396.

[2]Seban, R. A., and Faghri, A., "Evaporation and Heating with Turbulent Liquid Films," *Journal of Heat Transfer*, Transactions of American Society of Mechanical Engineers, Vol. 98, No. 2, 1976, pp. 315–318.

[3]Gorla, R. S. R., and T. M. Maloney, "Conjugate Heat Transfer in Falling Laminar Liquid Films," *Chemical Engineering Communications*, Vol. 60, No. 3, 1987, pp. 271–282.

[4]Gorla, R. S. R., and T. M. Maloney, "Conjugate Heat Transfer in Turbulent Falling Liquid Films," *Canadian Journal of Chemical Engineering*, Vol. 66, No. 3, 1988, pp. 334–337.

[5]Murthy, V. N., and P. K. Sarma, "Heat Transfer to Non-Newtonian Laminar Falling Liquid Films with Smooth Wave Free Gas-Liquid Interface," *International Journal of Multiphase Flow*, Vol. 4, No. 4, 1978, pp. 413–425.

[6]Stucheli, A., and F. Widmer, "Nicht-Isotherme riesel-filmstromung einer hochviskosen flussigkeit mit stork temperaturabhangiger viskositat," warme-und stoffubertragung, Vol. 11, No. 1, 1978, pp. 91–101.

Approximate Solution of the Thermal-Entry-Length Fluid Flow and Heat Transfer Characteristics in Annuli with Blowing at the Walls

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Nomenclature

C_P = specific heat
D_h = hydraulic diameter, $= 2(R_o - R_i)$
G = $(1 - K^2)/\ln(1/K)$
H = $(1 - K^4)/(1 - K^2)$
h = convective heat transfer coefficient
K = ratio of the inner radius to the outer radius of the annulus, $= R_i/R_o$
k = thermal conductivity of the fluid
Nu = Nusselt number, $= h D_h/k$
P^+ = dimensionless pressure, $= (P - P_e)/\rho w_{em}^2$
P = pressure
Pr = Prandtl number, $= \mu C_p/k$
Re_i = injection radial Reynolds number at the inner wall, $= R_i v_i/\nu$
Re_o = injection radial Reynolds number at the outer wall, $= R_o v_o/\nu$
R_i = inner radius of the annulus
R_o = outer radius of the annulus
r = radial direction
r^+ = dimensionless radial distance, $= r/R_o$
T = temperature
v = radial component of velocity
v^+ = dimensionless radial velocity, $= R_o v/\nu$
W^+ = dimensionless axial velocity based on the local mean velocity, $= w^+/w_m^+$
w = axial component of velocity
w_{em} = mean value of fully developed velocity profile at entrance to heated length
w^+ = dimensionless axial velocity, $= w/w_{em}$
z = axial direction
z^+ = dimensionless axial distance, $= \nu z/R_o^2 w_{em}$
θ_l = dimensionless temperature, $= (T - T_e)/(T_l - T_e)$
θ_{ml} = dimensionless mean temperature
μ = absolute viscosity
ν = kinematic viscosity
ρ = density

Subscripts
e = entrance to the annulus
i = inner wall
l = dummy subscript: o, i
m = mixed mean
o = outer wall

Superscript
$+$ = dimensionless variable

Received March 14, 1990; revision received and accepted for publication July 26, 1990.

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Introduction

THIS Note is a continuation of an effort to study the convective heat transfer in annular porous passages.[1] The subject of fluid flow in annuli with blowing and suction at the walls has received attention recently because of applications of the concentric annular heat pipe.[2,3] The numerical solutions by Faghri[1] were obtained using finite difference methods for thermally and hydrodynamically developing laminar flow in annular passages with blowing and suction at the walls. In the present study, the thermal-entry-length heat transfer characteristics of annuli with blowing at the walls have been approximated by using the fully developed axial and radial velocity profiles that were obtained analytically and used in a simple numerical scheme to solve the energy equation. An equation for the pressure drop in annuli with blowing or impermeable walls is also derived by solving the axial momentum equation with the fully developed axial and radial velocity profiles.

Analysis

The nondimensional equations associated with the flow in an annulus with uniformly porous walls are the continuity, momentum, and energy equations, as presented for the case of a constant-property, laminar, incompressible, steady flow with no transverse pressure gradient or longitudinal diffusion:

$$r^+ \frac{\partial w^+}{\partial z^+} + \frac{\partial (r^+ v^+)}{\partial r^+} = 0 \tag{1}$$

$$w^+ \frac{\partial w^+}{\partial z^+} + v^+ \frac{\partial w^+}{\partial r^+} = -\frac{\partial P^+}{\partial z^+} + \frac{\partial^2 w^+}{\partial r^{+2}} + \frac{1}{r^+}\frac{\partial w^+}{\partial r^+} \tag{2}$$

$$w^+ \frac{\partial \theta}{\partial z^+} + v^+ \frac{\partial \theta}{\partial r^+} = \frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial r^{+2}} + \frac{1}{r^+}\frac{\partial \theta}{\partial r^+}\right] \tag{3}$$

The hydrodynamic boundary conditions with a fully developed velocity profile at the entrance to the heated length are

$$w^+(0, r^+) = \frac{2[1 - r^{+2} + G \ln(r^+)]}{(H - G)}$$

$$w^+(z^+, K) = w^+(z^+, 1) = 0$$

$$v^+(z^+, K) = \frac{Re_i}{K}$$

$$v^+(z^+, 1) = Re_o$$

$$P^+(0, r^+) = 0$$

where $Re_i > 0$ and $Re_o < 0$ are the injection radial Reynolds numbers at the inner and outer walls, respectively. $Re_i = Re_o = 0$ corresponds to an impermeable wall. The fully developed entrance velocity profile was analytically derived for annuli with impermeable walls.[4]

Consider the general thermal boundary condition set that corresponds to a step temperature increase or decrease at the inner and outer walls:

$$T = T_e \qquad \text{everywhere for } z^+ \le 0 \tag{4a}$$

$$T = T_i \qquad r^+ = K, \qquad z^+ > 0 \tag{4b}$$

$$T = T_o \qquad r^+ = 1, \qquad z^+ > 0 \tag{4c}$$